Girraween High School

Year 12 Mathematics Extension 2

2026 HSC Task 2

Student Number 3953 2344

Question	Mark	MEX12-1	MEX12-2	MEX12-3	MEX12-4	MEX12-5	MEX12-6	MEX12-7
1 to 6	/6				✓			✓
7 to 10	/4			✓				✓
11	/12			✓				✓
12	/13			✓				✓
13	/13			✓				✓
14	/16				✓			✓
15	/16				✓			✓
16	/17				✓			✓
Total	/97			/42	/55			/97

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1 – 10

Question 1

Let $w=-3+i$. What is the value of $\frac{20}{iw}$?

A) $-2-6i$

B) $-2+6i$

C) $2-6i$

D) $2+6i$

Question 2

The roots of $z^3-1=0$ in modulus argument form are

A) $\text{cis } 0, \text{cis } \frac{2\pi}{3}$

B) $\text{cis } 0, \text{ cis } \frac{\pi}{3}$

C) $\text{cis } 0, \text{cis } \frac{2\pi}{3}, \text{cis}\left(-\frac{2\pi}{3}\right)$

D) $\text{cis } 0, \text{cis } \frac{\pi}{3}, \text{cis}\left(-\frac{\pi}{3}\right)$

Examination continues next page…

Question 3

By De Moivre's theorem, the value of $(1+i)^{10}$ is:

A) Purely real

B) Purely imaginary

C) A real multiple of $(1+i)$

D) An imaginary multiple of $(1+i)$

Question 4

Diagram A shows the complex number z represented in the Argand plane.

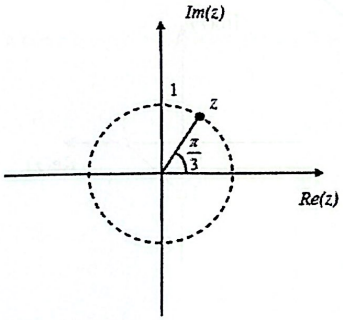


DIAGRAM A

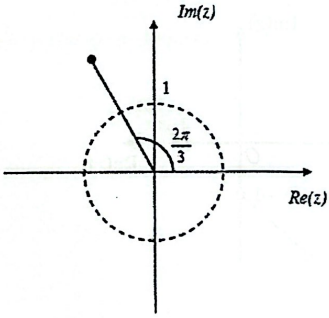


DIAGRAM B

Diagram B shows:

A) z^2

B) $2iz$

C) $-2z$

D) $2z^2$

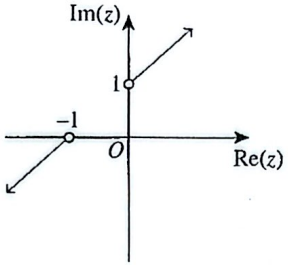
Examination continues next page...

Question 5

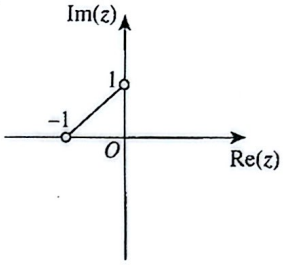
Which of the following Argand diagrams represents the solutions to the equation

$\text{Arg}(z-i) = \text{Arg}(z+1)$?

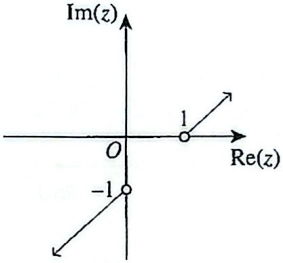
A.



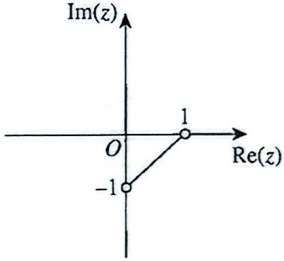
B.



C.



D.



Question 6

Which of the following is equivalent to $\dfrac{e^{-i\pi/2}}{e^{i\pi/6}}$

A) $-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

B) $-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

C) $\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

D) $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

Examination continues next page…

Question 7

Given $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 4\underset{\sim}{k}$, the angle between $\underset{\sim}{a}$ and positive $y-$axis is:

$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 3$

$3 = \sqrt{29}\cos\theta$

A) $\sin^{-1}\left(\frac{2}{\sqrt{29}}\right)$

B) $\cos^{-1}\left(\frac{3}{\sqrt{29}}\right)$

C) $\tan^{-1}\left(\frac{2}{3}\right)$

D) $\cot^{-1}\left(\frac{4}{3}\right)$

Question 8

If $\underset{\sim}{u} = 5\underset{\sim}{i} + (t-3)\underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} + (t+9)\underset{\sim}{j} + 2\underset{\sim}{k}$ are perpendicular, what are the possible values of t?

$\begin{pmatrix} 5 \\ t-3 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ t+9 \\ 2 \end{pmatrix} = 0$

$5 + (t-3)(t+9) + 6 = 0$

$11 + t^2 + 9t - 3t - 27 = 0$

$t^2 + 6t - 16 = 0$

$(t+8)(t-2) = 0$

A) $t = 8,\ 2$

B) $t = -8,\ -2$

C) $t = 8,\ -2$

D) $t = -8,\ 2$

Question 9

Find the equation of the line in cartesian form that passes through the point with position vector $2\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$ and is in the direction of $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$

$\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$2+$

$x = 2 + \lambda$

$y = -1 + 2\lambda$

A) $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-3}{-1}$

B) $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-1}$

C) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$

D) $\frac{x-5}{1} = \frac{y+1}{2} = \frac{z-5}{-1}$

Examination continues next page...

Question 10

The sketch of the curve in the shape of a saddle is drawn on the left. The curve has two peaks at $t = \frac{\pi}{4}, \frac{5\pi}{4}$ and two troughs at $t = \frac{3\pi}{4}, \frac{7\pi}{4}$, shown on the right.



Which of the following parametric equations are represented by the given curve:

A) $\begin{pmatrix} \sin t \\ \cos 2t \\ \sin t \end{pmatrix}$

B) $\begin{pmatrix} \sin t \\ \cos t \\ \sin 2t \end{pmatrix}$

C) $\begin{pmatrix} \sin 2t \\ \cos t \\ \sin t \end{pmatrix}$

D) $\begin{pmatrix} \sin 2t \\ \cos t \\ \sin 2t \end{pmatrix}$

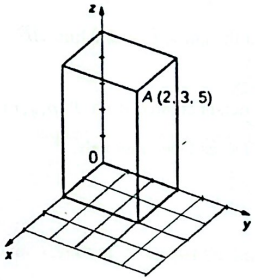
Section II (87 marks)

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) Start a new writing page

a) Consider a rectangular box with one of the vertices at the origin, as shown in the following figure.



If point A (2,3,5) is the opposite vertex to the origin, then find

(i) the coordinates of the other six vertices of the box. 3

(ii) the length of the diagonal of the box determined by the vertices O and A. 1

b) Find the vector parallel to $\underset{\sim}{u} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ that has a magnitude of 14. 2

c) Relative to a fixed origin O, the respective position vectors of three points A, B and C are $3\underset{\sim}{i} + 2\underset{\sim}{j} + 9\underset{\sim}{k}$, $-5\underset{\sim}{i} + 11\underset{\sim}{j} + 6\underset{\sim}{k}$ and $4\underset{\sim}{i} - 8\underset{\sim}{k}$.

(i) Determine the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2

(ii) Hence find, $\angle BAC$ correct to the nearest degree. 3

(iii) Calculate the area of ΔBAC 1

Examination continues next page…

Question 12 (13 marks) Start a new writing page

a) Given the points $A(3,4,-7)$ and $B(1,-1,6)$

(i) Find the vector equation of the line AB. 2

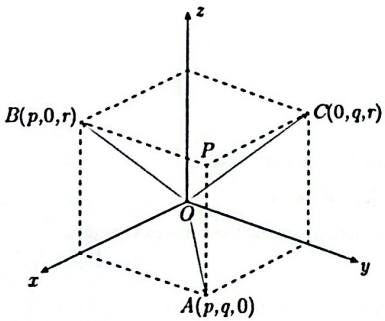
(ii) Reduce the equation in **(i)** to Cartesian form (Symmetric form). 3

b) Let P, A and B be the points $(3,-2,1)$, $(1,-11,-4)$ and $(9,3,8)$ respectively.

(i) Find $proj_{\underset{\sim}{b}}\underset{\sim}{p}$ where $\underset{\sim}{p}=\overrightarrow{AP}$ and $\underset{\sim}{b}=\overrightarrow{AB}$. 2

(ii) Hence calculate the perpendicular distance d from the point P to the line AB. 2

(c) The diagram below shows a rectangular prism in the first octant created from $P(p,q,r)$ with all edges parallel to the axes and with OA, OB and OC as face diagonals.



It is known that $\angle AOB=\angle AOC=\angle BOC=\frac{\pi}{3}$.

(i) Use dot products to show that $\dfrac{p^2}{\sqrt{p^2+q^2}\times\sqrt{p^2+r^2}}=\dfrac{q^2}{\sqrt{p^2+q^2}\times\sqrt{q^2+r^2}}$ 2

(ii) Hence prove that the rectangular prism is a cube (Hint: show $p=q$). 2

Examination continues next page...

Question 13 (13 marks) Start a new writing page

a) Find the point of intersection of the lines 4

$$\underset{\sim}{r} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -8 \\ 10 \end{pmatrix} \text{ and } \underset{\sim}{r} = \begin{pmatrix} -1 \\ -10 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ 7 \\ -2 \end{pmatrix}$$

b) **(i)** Find the vector equation of the sphere $x^2 + y^2 + z^2 - 4x - 4y - 2z = 0$ 3

(ii) The line l has equation $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}, \lambda \in \Re$ .Determine whether the line is a

tangent to the sphere or not. 3

c) Find the values of p so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \text{ and } \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$ are at right angles. 3

Examination continues next page…

Question 14 (16 marks) Start a new writing page

a) If $z=(1-i)(2\sqrt{3}+2i)$

(i) Express z in the form $x+iy$ where x and y are real. 2

(ii) By expressing $1-i$ and $2\sqrt{3}+2i$ in the modulus argument form, show that

$$z=4\sqrt{2}cis\left(-\frac{\pi}{12}\right)$$ 3

(iii) Hence find the exact value of $\tan\frac{\pi}{12}$ 3

b) Sketch the following on separate Argand diagrams.

(i) $\arg(z-(-1+2i))=-\frac{2\pi}{3}$ 2

(ii) $2<|z|\leq 4$ and $-\frac{\pi}{4}<\arg(z)\leq\frac{\pi}{2}$ 2

c) Find the values of θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number. 4

Examination continues next page...

Question 15 (16 marks) Start a new writing page

a) (i) If $z=\cos\theta+i\sin\theta$, show that $z^n+\frac{1}{z^n}=2\cos n\theta$ and $z^n-\frac{1}{z^n}=2i\sin n\theta$ 4

(ii) Use De Moivre's theorem to obtain an expression for $\sin^7\theta$ in the form

$a\sin 7\theta+b\sin 5\theta+c\sin 3\theta+d\sin\theta$. What are the values of a,b,c and d. 4

b) (i) Solve $z^6+1=0$. Show the roots on an Argand diagram. 4

(ii) Resolve z^6+1 into real quadratic factors. Hence show that

$$\cos 3\theta=4\left(\cos\theta-\cos\frac{\pi}{2}\right)\left(\cos^2\theta-\cos^2\frac{\pi}{6}\right)$$ 4

Examination continues next page...

Question 16 (17 marks) Start a new writing page

a) (i) Find the roots of $z^7 = 1$. 2

(ii) Show that the roots can be written in the form $1, \omega, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6$ where $\omega = cis\frac{2\pi}{7}$ 2

(iii) Show that $1+\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6 = 0$ 2

(iv) Form a quadratic equation with roots $\omega+\omega^2+\omega^4$ and $w^3+\omega^5+\omega^6$. 3

(v) Solve the quadratic equation in (iv). 1

(vi) Hence, find the exact value of $\sin\frac{2\pi}{7}+\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}$. 3

b) (i) Sketch $|z-2i|=1$ on an Argand diagram. 1

(ii) Find the maximum and minimum values of $\arg(z)$, $-\pi < \arg(z) \le \pi$ where z is a point on $|z-2i|=1$ 2

(iii) Find the value of z when $\arg(z)$ takes the maximum value and express in modulus - argument form. 1

End of Examination.

Mathematics Assessment Solutions

Girraween High School

page

❑ Year 11	❑ Extension 1	Year 2026
❑ Year 12	☑ Extension 2	Task Number 2

Suggested solutions | **Marker's Feedback**

Multiple choice

1. $\frac{20}{iw} = \frac{20}{i(-3+i)}$

$= \frac{20}{-3i-1} \times \frac{-1+3i}{-1+3i}$

$= -2+6i$ (B)

2. $\text{Cis } 0, \text{Cis } \frac{2\pi}{3}, \text{Cis}\left(-\frac{2\pi}{3}\right)$ (C)

3. $z = 1+i$

$= \sqrt{2}\,\text{cis}\,\frac{\pi}{4}$

$z^{10} = \left(\sqrt{2}\,\text{cis}\,\frac{\pi}{4}\right)^{10}$

$= (\sqrt{2})^{10}\,\text{cis}\,\frac{10\pi}{4}$

$= 2^5\,\text{cis}\,\frac{\pi}{2}$

$= 32i$ (B)

4 D

5 A

Suggested solutions	Marker's Feedback

6 $$\frac{e^{-\frac{i\pi}{2}}}{e^{\frac{i\pi}{6}}} = e^{\frac{-4i\pi}{6}} = e^{-\frac{2i\pi}{3}}$$

$$= \cos\left(\frac{2\pi}{3}\right) - i\sin\left(\frac{2\pi}{3}\right)$$

$$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$ (A)

7. The angle ~~θ~~ with positive y-axis is given by

$$\cos\theta = \frac{A_y}{|\underset{\sim}{a}|} = \frac{3}{\sqrt{29}}$$ (B)

8. $\underset{\sim}{y} \cdot \underset{\sim}{v} = 0$

$$t^2 + 6t - 16 = 0$$

$$(t+8)(t-2) = 0$$

$$t = -8, 2$$ (D)

9. $(2, -1, 4)$ satisfies

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

direction vector is $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

(C)

Suggested solutions	Marker's Feedback

10. when $t = \frac{\pi}{4}$

A) $x = \frac{1}{\sqrt{2}}$ $y = 0$ $z = \frac{1}{\sqrt{2}}$ ✗

B) $x = \frac{1}{\sqrt{2}}$ $y = \frac{-1}{\sqrt{2}}$ $z = 1$ } peak

$t = \frac{5\pi}{4}$

$x = \frac{-1}{\sqrt{2}}$ $y = \frac{-1}{\sqrt{2}}$ $z = 1$ } peak

$t = \frac{3\pi}{4}$

$x = \frac{1}{\sqrt{2}}$ $y = \frac{-1}{\sqrt{2}}$ $z = -1$ } troughs

$t = \frac{7\pi}{4}$

$x = \frac{-1}{\sqrt{2}}$ $y = \frac{1}{\sqrt{2}}$ $z = -1$ } troughs

(B)

Question 11 (12 marks)

(a)(i) The coordinates are

(2,0,5) (2,0,0) (2,3,0) (3 marks)

(0,3,0) (0,3,5) (0,0,5)

Done well

(ii) $OA = \sqrt{4+9+25}$

$= \sqrt{38}$ (1 mark)

Suggested solutions	Marker's Feedback

b) $|\underset{\sim}{y}| = \sqrt{2^2+1^2+2^2} = \sqrt{9} = 3$

Unit vector along the direction of

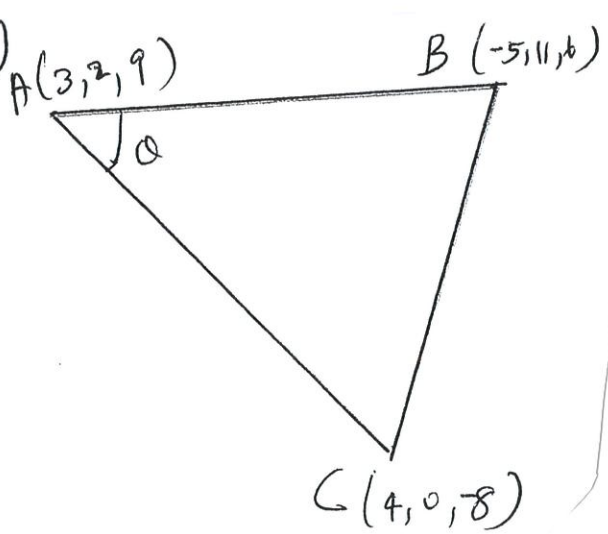
$\underset{\sim}{y} = \dfrac{2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}}{3}$ 1

(2 marks)

Parallel vector $= \dfrac{14(2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k})}{3}$ 1

Done well

c)



(i) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$= \begin{pmatrix} -5 \\ 11 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 9 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \\ -3 \end{pmatrix}$ 1

(2 marks)

$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$= \begin{pmatrix} 4 \\ 0 \\ -8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 9 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -17 \end{pmatrix}$ 1

Done well

Suggested solutions	Marker's Feedback

(ii) $\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{AB}||\overrightarrow{AC}| \cos\theta$

$= (-8 \times 1) + (9 \times -2) + (-3 \times -17)$

$= 25$ 1

$|\overrightarrow{AB}| = \sqrt{64+81+9} = \sqrt{154}$

$|\overrightarrow{AC}| = \sqrt{1+4+289} = \sqrt{294}$ 1

$25 = \sqrt{154}\sqrt{294}\cos\theta$ (3 marks)

$\cos\theta = \dfrac{25}{\sqrt{154}\sqrt{294}} = 83^\circ$ 1

Done well

(iii) Area $= \frac{1}{2} \times \sqrt{154} \times \sqrt{294} \times \sin 83^\circ$

$= 105.60$ square units.

(1 mark)

Question 12 (13 marks)

(a) (i) A (3, 4, −7) B (1, −1, 6)

$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$= (1-3, -1-4, 6+7) = (-2, -5, 13)$

The vector equations of the line is

$\underset{\sim}{r} = (3, 4, -7) + \lambda(-2, -5, 13)$ 1 1

or (2 marks)

$\underset{\sim}{r} = (1, -1, 6) + \mu(-2, -5, 13)$

where $\lambda, \mu \in \mathbb{R}$

Suggested solutions	Marker's Feedback

(ii) $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 4 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -5 \\ 13 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -5 \\ 13 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3-2\lambda \\ 4-5\lambda \\ 13\lambda - 7 \end{pmatrix}$ 1

$x = 3 - 2\lambda \qquad \lambda = \frac{x-3}{-2}$

$y = 4 - 5\lambda \qquad \lambda = \frac{4-y}{5} = \frac{y-4}{-5}$ 1

$z = 13\lambda - 7$

$\lambda = \frac{z+7}{13}$

Cartesian equation is

$$\frac{x-3}{-2} = \frac{y-4}{-5} = \frac{z+7}{13}$$ 1

Dow well

The cartesian equation corresponding to $\underset{\sim}{r} = (1,-1,6) + \mu(-2,-5,13)$ is

3marks

$$\frac{x-1}{-2} = \frac{y+1}{-5} = \frac{z-6}{13}$$

Note students need to write only one solution.

Suggested solutions	Marker's Feedback

b (i) $P(3,-2,1)$ $A(1,-11,-4)$ $B(9,3,8)$

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

$= (3-1, -2+11, 1+4) = (2,9,5)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (9-1, 3+11, 8+4) = (8, 14, 12)$

$$\text{Proj}_{\underset{\sim}{b}} \underset{\sim}{p} = \frac{\underset{\sim}{p} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \cdot \underset{\sim}{b}$$

$\underset{\sim}{p} = \begin{pmatrix} 2 \\ 9 \\ 5 \end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix} 8 \\ 14 \\ 12 \end{pmatrix}$

$\underset{\sim}{p} \cdot \underset{\sim}{b} = 16 + 126 + 60 = 202$

$\underset{\sim}{b} \cdot \underset{\sim}{b} = 8^2 + 14^2 + 12^2 = 404$] 1

$$\text{Proj}_{\underset{\sim}{b}} \underset{\sim}{p} = \frac{202}{404}(8\underset{\sim}{i} + 14\underset{\sim}{j} + 12\underset{\sim}{k})$$

$$= \frac{1}{2}(8\underset{\sim}{i} + 14\underset{\sim}{j} + 12\underset{\sim}{k})$$

$$= 4\underset{\sim}{i} + 7\underset{\sim}{j} + 6\underset{\sim}{k}$$ 1

(2 marks)

Some got confused with $\underset{\sim}{p} = \begin{pmatrix} 2 \\ 9 \\ 5 \end{pmatrix}$ and $P(3,-2,1)$

Suggested solutions	Marker's Feedback

(ii) Perpendicular distance

$= \left| \text{Proj}_{\underset{\sim}{b}} \underset{\sim}{p} - \underset{\sim}{p} \right|$

$= \left| (4\underset{\sim}{i} + 7\underset{\sim}{j} + 6\underset{\sim}{k}) - (2\underset{\sim}{i} + 9\underset{\sim}{j} + 5\underset{\sim}{k}) \right|$ 1

$= \left| 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k} \right| = \sqrt{4+4+1}$

$= 3$ units 1 (2 marks)

Done well

C (i) $\overrightarrow{OA} \cdot \overrightarrow{OB} = |\overrightarrow{OA}|\,|\overrightarrow{OB}| \cos\frac{\pi}{3}$

$\begin{pmatrix} p \\ q \\ 0 \end{pmatrix} \cdot \begin{pmatrix} p \\ 0 \\ r \end{pmatrix} = \sqrt{p^2+q^2} \cdot \sqrt{p^2+r^2} \cos\frac{\pi}{3}$

$p^2 = \sqrt{p^2+q^2} \cdot \sqrt{p^2+r^2} \times \frac{1}{2}$

$\frac{1}{2} = \cos\frac{\pi}{3} = \frac{p^2}{\sqrt{p^2+q^2} \cdot \sqrt{p^2+r^2}}$ 1 —— (1)

$\overrightarrow{OA} \cdot \overrightarrow{OC} = |\overrightarrow{OA}|\,|\overrightarrow{OC}| \cos\frac{\pi}{3}$

$\begin{pmatrix} p \\ q \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ q \\ r \end{pmatrix} = \sqrt{p^2+q^2} \cdot \sqrt{q^2+r^2} \times \frac{1}{2}$

$\frac{1}{2} = \cos\frac{\pi}{3} = \frac{q^2}{\sqrt{p^2+q^2} \cdot \sqrt{q^2+r^2}}$ 1 —— (2)

Suggested solutions	Marker's Feedback

From ① and ② we get (2 marks)

$$\frac{p^2}{\sqrt{p^2+q^2}\cdot\sqrt{p^2+r^2}} = \frac{q^2}{\sqrt{p^2+q^2}\cdot\sqrt{q^2+r^2}}$$

Many got upto this point.

(ii) $$\frac{p^2}{\sqrt{p^2+r^2}} = \frac{q^2}{\sqrt{q^2+r^2}}$$

$$p^2\sqrt{q^2+r^2} = q^2\sqrt{p^2+r^2}$$

$$p^4(q^2+r^2) = q^4(p^2+r^2)$$

$$p^4q^2 + p^4r^2 - q^4p^2 - q^4r^2 = 0$$

$$p^4q^2 - q^4p^2 + p^4r^2 - q^4r^2 = 0$$

$$p^2q^2(p^2-q^2) + r^2(p^4-q^4) = 0$$

$$p^2q^2(p^2-q^2) + r^2\left[(p^2+q^2)(p^2-q^2)\right] = 0$$

$$(p^2-q^2)\left[p^2q^2 + r^2(p^2+q^2)\right] = 0$$

This is a challenging question. Many made mistakes in factorising the expression.

$p^2 - q^2 = 0$ (2 marks)

$p^2 = q^2$

$p = q$ Similarly we can show that $p=r$ and $q=r$. ∴ the rectangular prism is a cube.

Suggested solutions	Marker's Feedback

Question 13 (13 marks)

(a) If the given lines intersect, there must be two values of λ and μ such that

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -8 \\ 10 \end{pmatrix} = \begin{pmatrix} -1 \\ -10 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ 7 \\ -2 \end{pmatrix}$$

Most student did this well.

$$\begin{pmatrix} 2+6\lambda \\ 1-8\lambda \\ -2+10\lambda \end{pmatrix} = \begin{pmatrix} -1+6\mu \\ -10+7\mu \\ 5-2\mu \end{pmatrix}$$

$2+6\lambda = 6\mu - 1$

$1-8\lambda = 7\mu - 10$

$-2+10\lambda = 5-2\mu$

$6\lambda - 6\mu = -3$

$-8\lambda - 7\mu = -11$

$10\lambda + 2\mu = 7$

$2\lambda - 2\mu = -1$ — ①

$-8\lambda - 7\mu = -11$ — ②

$10\lambda + 2\mu = 7$ — ③

① + ③

$12\lambda = 6 \quad \lambda = \frac{1}{2}$

$2\mu = 7 - 10\lambda$

$= 2$

$\mu = 1$ | |

$\lambda = \frac{1}{2}, \mu = 1$ |

Substitute in ②

LHS $= -8\lambda - 7\mu$

$= -8 \times \frac{1}{2} - 7 \times 1$

$= -11$

RHS $= -11$

LHS = RHS |

Some students forgot to check in the third equation & lost a mark.

Remember: you must use the SAME TWO EQUATIONS to FIND ~~the~~ ~~point~~ ~~λ and μ~~ for the point then CHECK in the equation you didn't use.

Suggested solutions	Marker's Feedback

Point of intersection is given by

$$\begin{pmatrix} 2+6\lambda \\ 1-8\lambda \\ -2+10\lambda \end{pmatrix} = \begin{pmatrix} 2+6\times\frac{1}{2} \\ 1-8\times\frac{1}{2} \\ -2+10\times\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 3 \end{pmatrix} \quad 1$$

(4 marks)

OR

$$\begin{pmatrix} -1+6\mu \\ -10+7\mu \\ 5-2\mu \end{pmatrix} = \begin{pmatrix} -1+6 \\ -10+7 \\ 5-2 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 3 \end{pmatrix}$$

b (i) $x^2-4x+y^2-4y+z^2-2z=0$

$x^2-4x+4+y^2-4y+4+z^2-2z+1$

$= 4+4+1$

$(x-2)^2+(y-2)^2+(z-1)^2=9$

Centre $(2,2,1)$ 1 radius $=3$ 1

The vector equation of the sphere is $\left|\underset{\sim}{r}-\begin{pmatrix}2\\2\\1\end{pmatrix}\right|=3$ 1

where $\underset{\sim}{r}$ is the position vector of a variable point on the sphere.

(3 marks)

Generally well done.

Some silly mistakes.

Suggested solutions	Marker's Feedback

(ii) If the line is a tangent, then only one value of λ exists for

$$\left| \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \right| = 3$$

$$\left| \begin{pmatrix} 3-\lambda \\ 6+2\lambda \\ -2\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \right| = 3$$

$$\left| \begin{pmatrix} 3-\lambda-2 \\ 6+2\lambda-2 \\ -2\lambda-1 \end{pmatrix} \right| = 3$$

$$\left| \begin{pmatrix} 1-\lambda \\ 4+2\lambda \\ -2\lambda-1 \end{pmatrix} \right| = 3 \quad 1$$

$(1-\lambda)^2 + (4+2\lambda)^2 + (-2\lambda-1)^2 = 9$ → Can also just show $\Delta = 0$ & not continue to finding part of intersection. Some students did this.

$(1-\lambda)^2 + (4+2\lambda)^2 + (2\lambda+1)^2 = 9$

i.e. $9\lambda^2 + 18\lambda + 9 = 0$

$\lambda^2 + 2\lambda + 1 = 0$ 1

3 marks

$(\lambda+1)^2 = 0$

$\lambda = -1$ There is only one solution for the equation

$\lambda^2 + 2\lambda + 1 = 0$. ∴ the line is a tangent to the sphere. 1

Suggested solutions	Marker's Feedback

(c) Writing the lines in standard form we get

$$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2} \quad \text{—①}$$ 1

$$\frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5} \quad \text{—②}$$

Since the lines are perpendicular, the scalar product of the direction vectors of ① and ② is zero.

$$\begin{pmatrix} -3 \\ \frac{2p}{7} \\ 2 \end{pmatrix} \cdot \begin{pmatrix} \frac{-3p}{7} \\ 1 \\ -5 \end{pmatrix} = 0 \quad 1$$

$$-3 \times \frac{-3p}{7} + \frac{2p}{7} \times 1 + 2 \times -5 = 0$$

$$\frac{9p}{7} + \frac{2p}{7} - 10 = 0$$

$$\frac{11p}{7} = 10$$

3 marks

$$p = \frac{70}{11} \quad 1$$

Generally well done.
A lot of silly mistakes finding p.

However, a small minority of studats didn't understand that DIRECTION VECTORS ONLY matter.

Suggested solutions	Marker's Feedback

Question 14 (16 marks)

(a)(i) $z = (1-i)(2\sqrt{3}+2i)$ (2 marks)

$= 2\sqrt{3} + 2i - i2\sqrt{3} - 2i^2$

$= 2 + 2\sqrt{3} + i(2 - 2\sqrt{3})$ —— (1)

(ii) $z = 1 - i$

$|z| = \sqrt{2}$

$\tan\alpha = 1,\ \alpha = \frac{\pi}{4},\ \theta = -\frac{\pi}{4}$

$1 - i = \sqrt{2}\ \text{cis}\left(\frac{-\pi}{4}\right)$ 1

$w = 2\sqrt{3} + 2i$

$|w| = \sqrt{12+4} = 4$

$\tan\alpha = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},\ \alpha = \frac{\pi}{6},\ \theta = \frac{\pi}{6}$

$2\sqrt{3} + 2i = 4\,\text{cis}\,\frac{\pi}{6}$ 1

$(1-i)(2\sqrt{3}+2i) = \left(\sqrt{2}\ \text{cis}\,\frac{-\pi}{4}\right)\left(4\,\text{cis}\,\frac{\pi}{6}\right)$

$= 4\sqrt{2}\ \text{cis}\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$

$= 4\sqrt{2}\ \text{cis}\left(\frac{-\pi}{12}\right)$ 1 (3 marks)

$= 4\sqrt{2}\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)$

$= 4\sqrt{2}\cos\frac{\pi}{12} - i\,4\sqrt{2}\sin\frac{\pi}{12}$ —— (2)

Generally very well done which is important as this is a STANDARD Ext 2 question.

Suggested solutions	Marker's Feedback

(iii) Equating real and imaginary parts of (1) and (2) we get.

$$4\sqrt{2}\cos\frac{\pi}{12} = 2+2\sqrt{3}$$

$$\cos\frac{\pi}{12} = \frac{2+2\sqrt{3}}{4\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}} \quad |$$

$$-4\sqrt{2}\sin\frac{\pi}{12} = 2-2\sqrt{3}$$

$$\sin\frac{\pi}{12} = \frac{2-2\sqrt{3}}{-4\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} \quad |$$

Generally well done.

$$\tan\frac{\pi}{12} = \frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{1+\sqrt{3}}$$

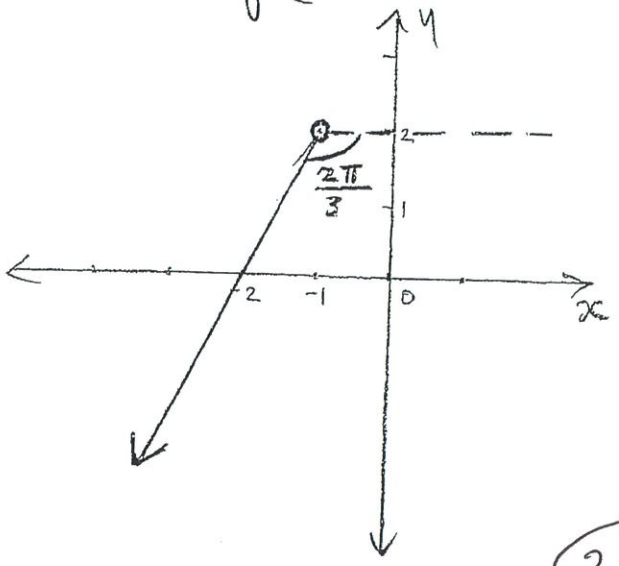
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3} \quad | \quad \text{(3 marks)}$$

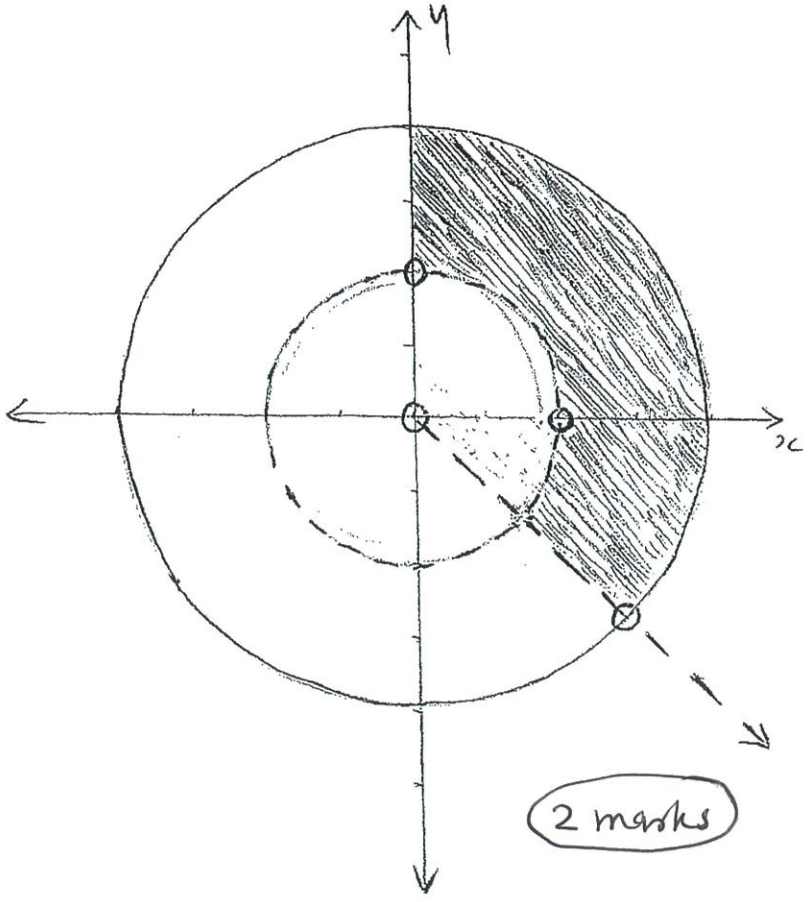
Suggested solutions	Marker's Feedback

(b) (b) (i) $\arg\left(z-(-1+2i)\right) = -\frac{2\pi}{3}$



2 marks

(ii) $2 < |z| \leq 4$ and $-\frac{\pi}{4} \leq \arg z \leq \frac{\pi}{2}$



2 marks

Generally well done [and should be as a STANDARD Ext 2 question).

Suggested solutions	Marker's Feedback

(c) $\dfrac{3+2i\sin\theta}{1-2i\sin\theta} = \dfrac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)}$ |

$= \dfrac{3+6i\sin\theta+2i\sin\theta+4i^2\sin^2\theta}{1-4i^2\sin\theta}$

$= \dfrac{3+8i\sin\theta-4\sin^2\theta}{1+4\sin^2\theta}$

$= \dfrac{3-4\sin^2\theta}{1+4\sin^2\theta} + i \times \dfrac{8\sin\theta}{1+4\sin^2\theta}$ |

Since $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real,

its imaginary part is zero.

$\dfrac{8\sin\theta}{1+4\sin^2\theta} = 0$ |

$8\sin\theta = 0$ (4 marks)

$\sin\theta = 0$

$\theta = 0, \pi, 2\pi$ |

Some students oversimplified this & jumped to $\sin\theta = 0$ without showing ANY working & lost marks.

Suggested solutions	Marker's Feedback

Question 15 (16 marks)

(a) (i) $z = \cos\theta + i\sin\theta$

$z^n = (\cos\theta + i\sin\theta)^n$

$= \cos n\theta + i\sin n\theta$ by De Moivre's theorem

$\frac{1}{z^n} = z^{-n} = (\cos\theta + i\sin\theta)^{-n}$

$= \cos(-n\theta) + i\sin(-n\theta)$ by DeMoivre's theorem

$= \cos n\theta - i\sin n\theta$ $\left(\because \sin(-\theta) = -\sin\theta,\ \cos(-\theta) = \cos\theta\right)$

Done well

$z^n + \frac{1}{z^n} = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$

$= 2\cos n\theta$ (4 marks)

$z^n - \frac{1}{z^n} = \cos n\theta + i\sin n\theta - \cos n\theta + i\sin n\theta$

$= 2i\sin n\theta$

(ii) $(2i\sin\theta)^7 = \left(z - \frac{1}{z}\right)^7$

$= z^7 - 7z^5 + 21z^3 - 35z + 35\times\frac{1}{z} - 21\times\frac{1}{z^3} + 7\times\frac{1}{z^5} - \frac{1}{z^7}$

Some got confused with the expansion of $\sin^7\theta$ and $\sin^7\theta$

$= \left(z^7 - \frac{1}{z^7}\right) - 7\left(z^5 - \frac{1}{z^5}\right) + 21\left(z^3 - \frac{1}{z^3}\right) - 35\left(z - \frac{1}{z}\right)$

$= 2i\sin 7\theta - 7\times 2i\sin 5\theta + 21\times 2i\sin 3\theta - 35\times 2i\sin\theta$

Suggested solutions	Marker's Feedback

$$2^7 i^7 \sin^7\theta = 2i\sin 7\theta - 14i\sin 5\theta + 42i\sin 3\theta - 70i\sin\theta$$

$$-64\sin^7\theta = \sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta$$

$$\sin^7\theta = \frac{-1}{64}\left[\sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta\right]$$

Made mistakes in calculating the powers of i

$$= \frac{-1}{64}\sin 7\theta + \frac{7}{64}\sin 5\theta - \frac{21}{64}\sin 3\theta + \frac{35}{64}\sin\theta$$ (4 marks)

$$a = \frac{-1}{64} \quad b = \frac{7}{64} \quad c = \frac{-21}{64} \quad d = \frac{35}{64}$$

(b)(i) $z^6 + 1 = 0$

$$z^6 = -1 = \cos\pi + i\sin\pi$$

$$= \cos(2k+1)\pi + i\sin(2k+1)\pi, \quad k \in \mathbb{Z}$$

$$z = \left[\cos(2k+1)\pi + i\sin(2k+1)\pi\right]^{\frac{1}{6}}$$

Done well

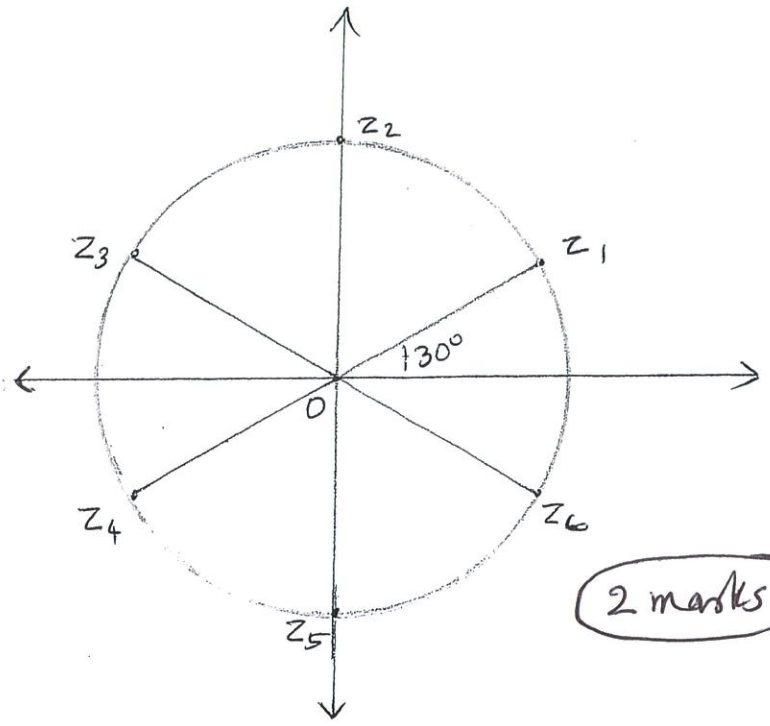
$$= \cos\frac{(2k+1)\pi}{6} + i\sin\frac{(2k+1)\pi}{6}$$

$k = 0, 1, 2, 3, 4, 5$

(by De Moivre's theorem)

(2 marks)

Suggested solutions	Marker's Feedback



2 marks

Done well

(ii) From the diagram

$z_6 = \overline{z_1}$, $z_5 = \overline{z_2}$, $z_4 = \overline{z_3}$

$z_1 + z_6 = z_1 + \overline{z_1} = 2\cos\frac{\pi}{6}$ 1

$z_1 z_6 = z_1 \overline{z_1} = 1$

$z_3 + z_4 = z_3 + \overline{z_3} = 2\cos\frac{5\pi}{6}$

$z_3 z_4 = z_3 \overline{z_3} = 1$

Mostly done well

$z^6 + 1 = (z - z_1)(z - z_6)(z - z_3)(z - z_4)(z - i)(z + i)$

$= (z^2 + 1)\left(z^2 - 2\cos\frac{\pi}{6}z + 1\right)\left(z^2 - 2\cos\frac{5\pi}{6}z + 1\right)$ 1

Suggested solutions	Marker's Feedback

divide both sides by z^3

$$\frac{z^6+1}{z^3} = \frac{z^2+1}{z} \times \frac{z^2-2\cos\frac{\pi}{6}z+1}{z} \times \frac{z^2-2\cos\frac{5\pi}{6}z+1}{z} \quad |$$

$$z^3+\frac{1}{z^3} = \left(z+\frac{1}{z}\right)\left(z-2\cos\frac{\pi}{6}+\frac{1}{z}\right)\left(z-2\cos\frac{5\pi}{6}+\frac{1}{z}\right)$$

$$2\cos 3\theta = 2\cos\theta\left(2\cos\theta-2\cos\frac{\pi}{6}\right)\left(2\cos\theta-2\cos\frac{5\pi}{6}\right)$$

$$2\cos 3\theta = 8\cos\theta\left(\cos\theta-\cos\frac{\pi}{6}\right)\left(\cos\theta-\cos\frac{5\pi}{6}\right)$$

$$\cos 3\theta = 4\cos\theta\left(\cos\theta-\cos\frac{\pi}{6}\right)\left(\cos\theta-\cos\frac{5\pi}{6}\right)$$

$$= 4\left(\cos\theta-\cos\frac{\pi}{2}\right)\left(\cos\theta-\cos\frac{\pi}{6}\right)\left(\cos\theta+\cos\frac{5\pi}{6}\right)$$

$$\left(\because \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6}\right) \quad |$$

4 marks

$$= 4\left(\cos\theta-\cos\frac{\pi}{2}\right)\left(\cos^2\theta-\cos^2\frac{\pi}{6}\right)$$

Many forgot to mention

$$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6}$$

Some didn't get the idea of dividing by z^3.

Suggested solutions	Marker's Feedback

Question 16 (17 marks)

(a) (i) $z^7 = 1 = \text{cis } 0 = \text{cis } 2k\pi$, $k \in \mathbb{Z}$

$z = [\text{cis } 2k\pi]^{\frac{1}{7}}$

$= \text{cis } \frac{2k\pi}{7}$ $k = 0, 1, 2, 3, 4, 5, 6$

by De Moivre's theorem

Done well

Roots are (2 marks)

$1, \text{cis } \frac{2\pi}{7}, \text{cis } \frac{4\pi}{7}, \text{cis } \frac{6\pi}{7}, \text{cis } \frac{8\pi}{7}, \text{cis } \frac{10\pi}{7}, \text{cis } \frac{12\pi}{7}$

(ii) Let $w = \text{cis } \frac{2\pi}{7}$

$w^2 = \left(\text{cis } \frac{2\pi}{7}\right)^2 = \text{cis } \frac{4\pi}{7}$

$w^3 = \left(\text{cis } \frac{2\pi}{7}\right)^3 = \text{cis } \frac{6\pi}{7}$

$w^4 = \left(\text{cis } \frac{2\pi}{7}\right)^4 = \text{cis } \frac{8\pi}{7}$

Done

(2 marks)

$w^5 = \left(\text{cis } \frac{2\pi}{7}\right)^5 = \text{cis } \frac{10\pi}{7}$

$w^6 = \left(\text{cis } \frac{2\pi}{7}\right)^6 = \text{cis } \frac{12\pi}{7}$

$\therefore$ the roots can be written as

$1, w, w^2, w^3, w^4, w^5$ and w^6

Suggested solutions	Marker's Feedback

(iii) $1+w+w^2+w^3+w^4+w^5+w^6$

$= -\dfrac{\text{Coefficient of } z^6}{\text{Coefficient of } z^7} = \dfrac{-0}{1} = 0$

(2 marks)

Must show for full marks (not just state sum of roots)

(iv) $z_1 = w+w^2+w^4, \quad z_2 = w^3+w^5+w^6$

$z_1+z_2 = w+w^2+w^4+w^3+w^5+w^6 = -1$

$z_1 z_2 = (w+w^2+w^4)(w^3+w^5+w^6)$

$= w^4+w^6+w^7+w^5+w^7+w^8+w^7$

$+w^9+w^{10}$

$= w^4+w^6+1+w^5+1+w+1+w^2+w^3$

$= 0+2 = 2$

The quadratic equation with roots z_1 and z_2 is

(3 marks)

$z^2-(-1)z+2=0$

$z^2+z+2=0$

Done well

(v) $z^2+z+2=0$

$z = \dfrac{-1\pm\sqrt{-7}}{2} = \dfrac{-1\pm i\sqrt{7}}{2}$

$= \dfrac{-1}{2}+\dfrac{i\sqrt{7}}{2}$ or $\dfrac{-1}{2}-\dfrac{i\sqrt{7}}{2}$ —(1)

Done well

Suggested solutions	Marker's Feedback

(vi) Consider $z_1 = w + w^2 + w^4$

$= \text{cis}\frac{2\pi}{7} + \text{cis}\frac{4\pi}{7} + \text{cis}\frac{8\pi}{7}$

$= \left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}\right) + i\left(\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}\right)$ ——(2)

Comparing (1) and (2) we get

$\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7} = \frac{\sqrt{7}}{2}$ or $-\frac{\sqrt{7}}{2}$ ——(3)

But $\sin\frac{4\pi}{7} = \sin\left(\pi - \frac{4\pi}{7}\right) = \sin\frac{3\pi}{7}$

$\sin\frac{8\pi}{7} = -\sin\left(\frac{8\pi}{7} - \pi\right) = -\sin\frac{\pi}{7}$

(3) becomes

$\sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{\pi}{7} = \frac{\sqrt{7}}{2}$ or $-\frac{\sqrt{7}}{2}$

$\sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{\pi}{7} > 0$

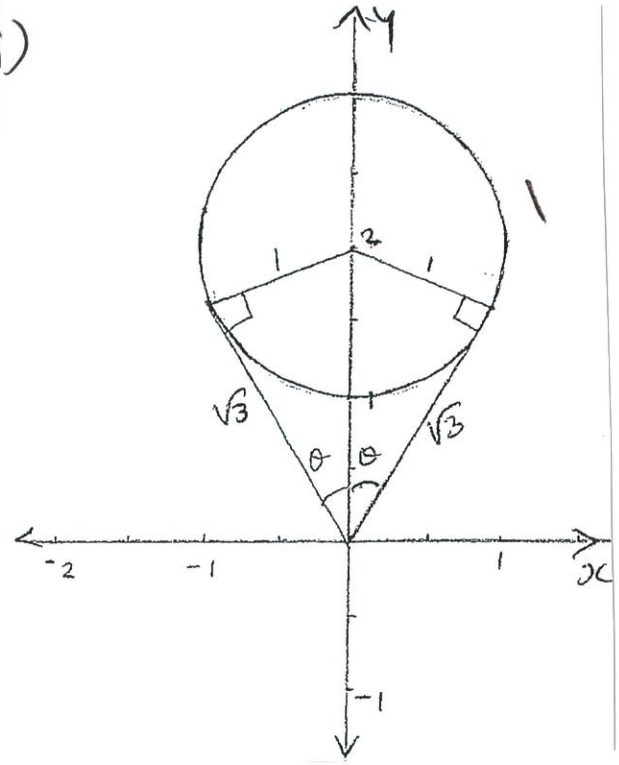
$\therefore \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{\pi}{7} = \frac{\sqrt{7}}{2}$

3 marks

Some did not know where to begin

Suggested solutions	Marker's Feedback

b(i)



1

Done well

(ii) $\tan\theta = \frac{1}{\sqrt{3}} \qquad \theta = \frac{\pi}{6}$

Minimum $\arg(z) = \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{\pi}{3}$ 1

Done well

Maximum $\arg(z) = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$ 1

(iii) $z = \sqrt{3}\ \mathrm{cis}\ \frac{2\pi}{3}$ 1